FLOW OF A MIXTURE OF RAREFIED GASES BETWEEN TWO PARALLEL PLATES WITH SINUSOIDAL CONCENTRATION DISTRIBUTION AT THE BOUNDARY

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We investigate the effect of the transverse velocity component at a gas–plate boundary on the flow regime of a two-component rarefied gas between two parallel plates with sinusoidal concentration distribution on the lower plate.

We consider the isothermal flow of a two-component rarefied gas between two parallel plates with a sinusoidal concentration distribution on the lower plate.

We choose the X and Y axes, respectively, along, and along the normal to the surface of the lower plate. The concentration on the upper plate (Y = d) is constant and equals c_d, and on the lower plate the concentration is of the form

$$c = c_0\left(1 + \alpha \sin \frac{2\pi X}{L}\right).$$

We assume the following quantities to be small: the ratio $(c_0 - c_d)/c_0$, the coefficient α, and the ratio of the mean free path of gas molecules to the wavelength L of the concentration variation. For $c_0 \neq c_d$ and $\alpha = 0$, there is a transverse motion of gas between the plates. For $\alpha \neq 0$ on the lower plate there is diffusion slip, as a result of which there arises slow macroscopic motion (the velocities u and v are small).

We seek the pressure, density, and concentration distributions in the form

$$p = p_0(1 + \xi), \quad \rho = \rho_0(1 + \sigma), \quad c = c_0(1 + \tau).$$

The quantities with subscripts zero correspond to the undisturbed lower plate ($\alpha = 0$, $c = c_0$), ξ, σ, and τ are small.

Disregarding products of small quantities, we write the system of equations [1]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial \xi}{\partial x} = \mu_* \Delta u, \tag{2}$$

$$\frac{\partial \xi}{\partial y} = \mu_* \Delta v, \tag{3}$$

$$\Delta \tau = 0, \tag{4}$$

$$\xi = \sigma + m\tau. \tag{5}$$

Here we have introduced the following notation:

$$x = \frac{2\pi X}{L}, \quad y = \frac{2\pi Y}{L}, \quad \mu_* = \frac{2\pi\mu}{p_0 L},$$

$$m = 1 - \frac{\rho_0 R_2 T}{p_0}.$$

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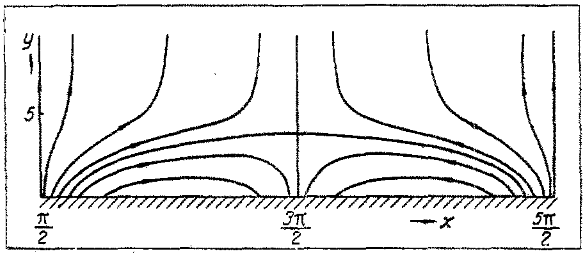

Fig. 1. Streamlines corresponding to the stream function (22).

We now state the boundary conditions. On the lower plate we must use a condition that takes account of diffusion slip [2, 3]. Furthermore, we consider the nonzero transverse velocity component at the gas – wall boundary [4].

Accordingly, the boundary conditions (in the same approximation as for the equation) are of the following form:

for y = 0

$$a_1\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+a_2\frac{\partial \tau}{\partial x}+a_3u=0, \tag{6}$$

$$-v(1-c_0)=a_4\frac{\partial \tau}{\partial y}, \tag{7}$$

$$\tau=\alpha\sin x, \tag{8}$$

where

$$a_1=-\frac{(2-f)\pi\mu}{fL},$$
$$a_2=-\frac{2\pi\rho_0D_{12}c_0}{L}\left(\frac{T}{2\pi}\right)^{1/2}\left(R_1^{1/2}-R_2^{1/2}\right),$$
$$a_3=\rho_0\left(\frac{T}{2\pi}\right)^{1/2}\left[c_0R_1^{1/2}+(1-c_0)R_2^{1/2}\right], \tag{9}$$
$$a_4=\frac{2\pi D_{12}c_0}{L};$$

for y = $2\pi d/L$, we use a condition similar to (6), and also the conditions

$$-v(1-c_d)=a_4\frac{\partial\tau}{\partial y}, \tag{10}$$

$$\tau=\frac{c_d-c_0}{c_0}. \tag{11}$$

Solving Eq. (4) with conditions (8) and (11) (assuming the quantity $\exp\{-2\pi d/L\}$ to be negligible), we obtain

$$\tau=\alpha\exp\{-y\}\sin x+\frac{L(c_d-c_0)}{2\pi dc_0}y. \tag{12}$$

Eliminating u and v from (1)-(3), we obtain the equation

$$\Delta\xi=0, \tag{13}$$

which has the solution

$$\xi=2\mu_*b_1\exp\{-y\}\sin x+2\mu_*b_2y. \tag{14}$$

The coefficients b_1 and b_2 will be determined below.

Using (12) and (14), we solve Eq. (2) with boundary conditions (7) and (10):

$$u=\exp\{-y\}\cos x\,(k-b_1y), \tag{15}$$
$$v=\exp\{-y\}\sin x\,(A_2+b_1y)+b_2y^2+A_1, \tag{16}$$

where

$$A_1=-\frac{a_4(c_d-c_0)L}{c_0(1-c_0)2\pi d}, \tag{17}$$

$$A_2=\frac{a_4\alpha}{1-c_0}, \tag{18}$$

$$k=\frac{-a_2\alpha}{a_3-2a_1}. \tag{19}$$

Substituting (15) and (16) into (1), we obtain

$$b_1 = k + A_2, \quad b_2 = 0.$$

Hence,

$$u = \exp\{-y\} \cos x \,[k - (k + A_2)\, y], \tag{20}$$

$$v = \exp\{-y\} \sin x \,[A_2 + (k + A_2)\, y] + A_1 \tag{21}$$

In the approximation assumed, the longitudinal velocity is zero on the upper plate.

On the basis of (17) and (18) we introduce the stream function

$$\Psi = \exp\{-y\} \cos x \,[A_2 + (k + A_2)\, y] - A_1 x. \tag{22}$$

The flow has period 2π. Using (18) and (19), we obtain $k + A_2 > 0$. If $c_d < c_0$, then $A_1 > 0$; for $c_d > c_0$, we have $A_1 < 0$. In the first case there is a singularity at $x = 3\pi/2,\ 7\pi/2$; in the second case, at $x = \pi/2,\ 5\pi/2$. Solution of the characteristic equation shows that the singularity is a saddle point. Figure 1 shows streamlines corresponding to (22), for $k < 0$.

As in [1], ξ is of second order of smallness, and therefore $\sigma \simeq -m\tau$.

If $c_0 = c_d$, then according to (17), we have $A_1 = 0$. In this case, both the longitudinal and transverse velocities equal zero on the upper plate.

Disregarding the effect of the transverse velocity component at the glass–wall boundary (i.e., replacing conditions (7) and (10) by the condition $v = 0$ for both plates), we obtain

$$u = k(1 - y) \exp\{-y\} \cos x,$$

$$v = ky \exp\{-y\} \sin x,$$

$$\Psi = ky \exp\{-y\} \cos x.$$

In this case the streamlines are closed curves with singularities of the "center" type, having coordinates $x = 0,\ \pi,\ 2\pi, \ldots;\ y = 1$. These results agree with the results obtained in [1] for the flow of a one-component rarefied gas with sinusoidal temperature distribution at the wall.

Thus, taking account of the effect of the transverse velocity component at the plate–gas boundary substantially changes the nature of such flows.

NOTATION

$\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$;	
p and ρ	are the gas pressure and density;
T	is the gas temperature;
u and v	are the longitudinal and transverse velocity components;
μ	is the coefficient of viscosity of the gas;
D_{12}	is the binary diffusion coefficient;
R_1 and R_2	are the gas constants of the components;
c	is the concentration of component 1;
f	is the accommodation coefficient.

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